

**Instructions:** Upload LEGIBLE, COMPLETE solutions to Gradescope before 11:59pm on 18 October 2021.

1. Either compute the following limit or show it does not exist.

$$\lim_{(x,y) \rightarrow (1,-1)} \frac{6x - y - 5}{\sqrt{(x-1)^2 - (y+1)^2}}$$

2. Consider the function  $f(x, y, z) = \frac{x+y}{z}$  and let  $\mathbf{v} = \frac{1}{\sqrt{3}} \langle 1, -1, 1 \rangle$  and  $p = (0, -3, 1)$ .

(a) Compute the directional derivative  $D_{\mathbf{v}}f(p)$  via the definition.

(b) Compute the gradient of  $f$  at  $p$ .

(c) Compute the directional derivative  $D_{\mathbf{v}}f(p)$  via the gradient formula.

3. Compute all second order partial derivatives of the function  $f(x, y, z) = \cos(x^2y) - z \arctan(xy)$ .

4. Let  $f(x, y) = \sin(xy^2 + xy)$  where  $x(r, s, t) = r^2 + s^2 + t^2$  and  $y(r, s, t) = rst$ . Compute  $\frac{\partial f}{\partial r}$ ,  $\frac{\partial f}{\partial s}$ , and  $\frac{\partial f}{\partial t}$  via the chain rule.

5. Compute  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$  for a function  $z(x, y)$  satisfying the equation  $(x^2 + y^2 + z^2 + 3)^2 = 16(x^2 + y^2)$ .

6. Compute the tangent plane to the function  $f(x, y, z) = \frac{x + \cos(\sqrt{y}z)}{xy}$  at  $p = (-1, \frac{1}{4}, \pi)$ .

7. Compute the total differential of  $f(x, y, z) = \frac{x+y}{z}$ .